Food freezing and thawing time prediction with new simple calculation formulas application

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Abstract

There were developed new methods for freezing and thawing time prediction analysing the effects of water behaviour and product properties. Introduction of a characteristic freezing temperature resulted in the elimination of cryoscopic temperature out of the calculations. Hence, both models got more universality. The models had been comprehensively verified and their high accuracy and full practicability was confirmed.

Nomenclature

A	- area, [m^2]
Bi	- Biot number
c	- specific heat capacity, [J/kg K]
d	- characteristic dimension, [m]
E	- prediction error, [%]
h	- heat transfer coefficient, [W/m2 K]
ΔH	- enthalpy difference, [J/m3]
k	- heat conductivity, [W/m K]
L	- latent heat of freezing, [J/kg]
R	- correlation coefficient
t	- time, [s, h]
T	- temperature, [K, ^{o}C]
V	- volume, [m^3]
ρ	- density, [kg/m^3]
ω	- degree of water freezing out, %

Indices

c	- centre, end
kr	- cryoscopic
n	- unfrozen
o	- medium
p	- initial
rz	- experimental
z	- frozen

Introduction

Particularly significant effects observed at food freezing-thawing processes are freezing (crystallization) - water melting in a product structure. The general character of water freezing out kinetics is most often described by logarithmic dependencies (Fig.1) [1]. Regarding the complexity of the changes from both, physical and biochemical aspect as well as their determination of process course (freezing, thawing) and product quality, the effects were and still have been the objects under investigation in the food technology and engineering and some fields related with them (i.e. cryobiology, cryosurgery). From an engineering and technological aspect the most significant fact is a way in which a process of water freezing out- ice melting in a product and product properties formed due to these and other factors operation affect the very process and product quality following the treatment. Thawing is an ultimate operation in a cooling chain performed directly before further industrial treatment or product relay for market.

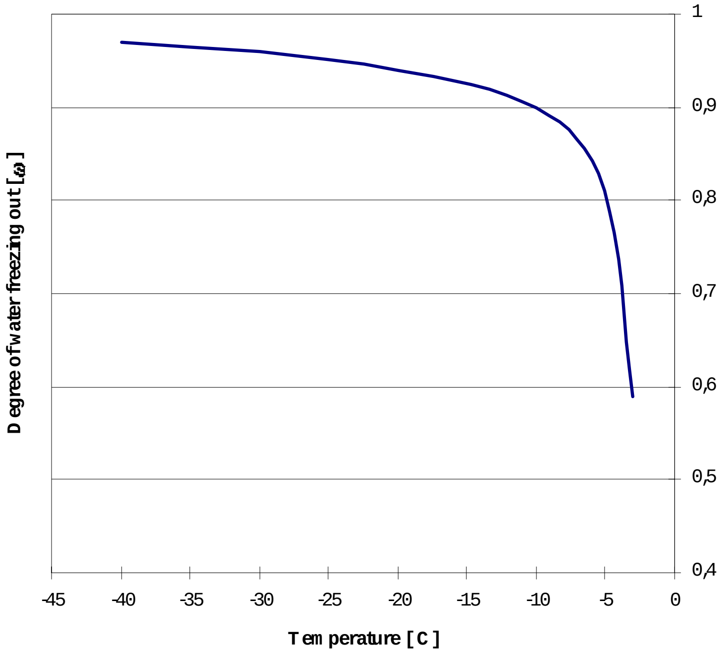

Figure 1: Dependency of water freezing out degree (ω) on product temperature (T).

This process aims at bringing the products to a state suitable for further use, in a maximal degree approaching a product natural state. Possibility for freezing and thawing time prediction allows, to form advantageous process course, to facilitate fixing effects as well as to enable a refrigerating machine design. Considering ice crystallisation and melting processes in a problem fundamental for food freezing and thawing, it was comprehensively examined to work out a new analytical and empirical methods those they would let predict effective freezing and thawing time. So, the objective of the investigation was to work out, simplified models for freezing and thawing time calculation for foods of regular dimensionality.

Development of the models

Some possibility to obtain the new simple freezing time prediction model [7] was found in Mott's method [1]. Calculations acc. to this method consist in the application of criterion numbers:

$$S = \frac{B+1}{G} = A \cdot \frac{d}{V}, \qquad B = \frac{h \cdot d}{2 \cdot \lambda_z}, \qquad G = \frac{t \cdot h \cdot \Delta T}{\rho_z \cdot Q \cdot d} \tag{1}$$

Comparison of the dependencies of Eq. (1) leads to the form:

$$t = \frac{Q}{\Delta T} \cdot \frac{V}{A} \cdot \left(\frac{d}{2 \cdot \lambda_z} + \frac{1}{h} \right) \tag{2}$$

Defining the ratio of product volume *V* to its surface *A* as a shape factor *K* and the ratio of the heat *Q* removed from the product to its temperature change *ΔT* as *ΔH,* the following dependence for the freezing time can be obtained:

$$t = \Delta H \cdot K \cdot \left(\frac{d}{2 \cdot \lambda_z} + \frac{1}{h} \right) \tag{3}$$

Because the thermal conductivity of most frozen food, is about 2.0 [W/m K], it can be assumed that $2\lambda_z = 4$ *[W/mK],* and finally the Eq. (3) becomes:

$$t = \Delta H \cdot K \cdot \left(\frac{d}{4} + \frac{1}{h} \right) \tag{4}$$

Considering the fact, the heat removed during the freezing process consists of the initial product enthalpy surplus ΔI_P in relation to the cryoscopic condition, latent heat of freezing L_F and its enthalpy change ΔI_T in subcooling, ΔH is defined as:

$$\Delta H = \Delta I_P + L_F + \Delta I_T \tag{5}$$

The final form of the dependence (5) is obtained by the experimental methods correcting particular terms and values in the equation because of the minimum error of freezing time calculations for the conditions under which our measurements were performed [4,7].

$$\Delta H = \rho_n c_n \cdot \frac{0{,}74\, T_p - T_F}{\frac{T_p}{2{,}4} - T_o} + \frac{\rho_z L}{T_F - T_o} + \rho_z c_z \frac{T_F - 0{,}5\left(T_c + T_o\right)}{T_F - T_o} \tag{6}$$

At the characteristic freezing temperature T_F the amount of the frozen water is 70% of the total water content in product. By some transformation of Riedel's results [1], the dependence of product temperature on the ice fraction can be expressed as:

$$T = -\left[10^{\frac{0{,}31 \cdot b}{1{,}105 - b}} - \left(1 - |T_{kr}| \right) \right] \tag{7}$$

Hence, the characteristic product freezing temperature is:

$$T_F = -\left[3{,}43 - \left(1 + T_{kr} \right) \right], \tag{8}$$

and because the freezing temperature is often about T_{kr} = -1.0 °C, then

$$T_F = -3.4 \ \ [^{o}C] \tag{9}$$

For a rectangular brick or a finite cylinder, the shape factor *K* defined in the Eq. (3), can be obtained directly from the product dimensions. For an infinite slab, an infinite cylinder or a sphere the determined values of *K* are: slab = d/2,75; inf. Cylinder = d/5,6; sphere = d/8,2 respectively. On the basis of experimental researches there was obtained an every product thawing course related to temperature conditioned on process time (Fig.2). The process comprised phases: I (A-B) initial preheating of a product, II (B-C) specific thawing, III (C-D) product reheating up to a temperature required.

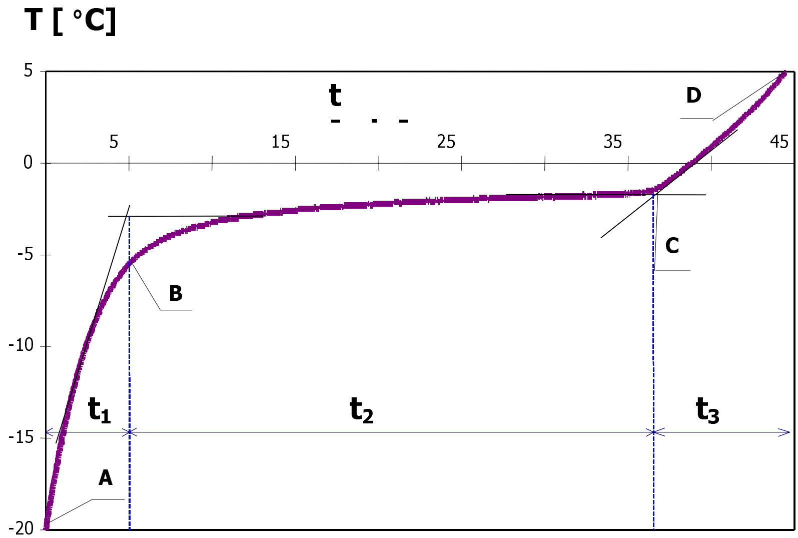

Figure 2: Typical temperature *versus* time profile for thawing a food product

The time of the first thawing phase was determined from the heat balance [3]:

$$V \cdot \rho \cdot c \cdot dT = -A \cdot h \cdot (T_o - T)dt \tag{10}$$

as

$$t_1 = \frac{\rho_z c_z V}{hA}\left(-\ln\frac{T_o - T_F}{T_o - T_p}\right) \tag{11}$$

The II-nd phase time was derived basing on the classical solution of the problem by Plank [1]:

$$t_2 = \frac{\rho_n LV}{hA(T_o + 3{,}4)}\left(1 + \frac{Bi}{2{,}8}\right) \tag{12}$$

Calculation formula for the product reheating time up to required final temperature was derived analogically as a dependence defining the first phase of the process

$$t_3 = \frac{\rho_n c_n V}{hA}\left(-\ln\frac{T_o - T_c}{T_o + 3{,}4}\right) \tag{13}$$

Considering that, the time of the complete thawing process is:

$$t = \frac{V}{hA}\left(C_z B + \frac{\rho_n L}{T_o + 3{,}4}E + C_n D\right) \tag{14}$$

where: C - specific heat capacity, B,D - temperature simplexes respectively, E - Biot number function.

Methods of model verification

The proposed model for freezing time prediction was verified due to its application for freezing time calculations, based on 383 freezing tests published, including six kinds of products shaped in five geometric forms [7]. The same calculations were performed by Cleland's and Pham's methods previously tested [8].

There were considered 232 published cases of the products thawing, the objects of regular shapes and broad range of process conditions. The selection of products was imposed by availability of reliable experimental data, which together with full characteristics of thawing parameters were taken from Cleland's works [5] and Ilicali [6]

The results of test predictions were statistically evaluated by: the characteristics of relative prediction errors, examining of fitting the results distribution to real values distribution, regression analysis of the experimental freezing and thawing time vs. the time predicted and also by one way variance analysis of relative prediction errors.

Results and discussion

Absolute mean value of relative prediction errors of the freezing time calculated by the new method, for the whole sample of 383 tests figures +1.74% (Tab. 1) and is found between the means of results obtained acc. to the Cleland's and Pham's methods.

Table 1: Statistical examination of the relative freezing time prediction errors (sample size 383)

	Average	Median	SD	SE	95% conf. int.
This work	1,74	2,61	12,40	0,64	0,49÷2,99
Pham	0,32	3,09	13,98	0,71	-1,08÷1,73
Cleland	-4,06	-4,54	13,60	0,69	-5,42÷2,69

The linear regression analysis was used to evaluate the convergence of the freezing time values obtained by the new method and the experimental data [Tab. 2] (Fig.3).

Table 2: Regression equations of the freezing time vs. the time predicted.

	this method	Pham	Cleland
Intercept *A*	0.0525	0.0398	0.0478
Slope *B*	0.9877	0.9950	0.9296
Correlation coef.	0.9902	0.9868	0.9896

To examine the distributions fitting of the freezing time values obtained by presented model and the experimental data Kolmogorov-Smirnov's test was used. This test confirmed the hypothesis H_o , that provided distribution compatibility of both variables (Fig.4).

Freezing times values obtained by the proposed method, those by Pham's and Cleland's methods were subjected to the one-way analysis of variance. As an examined parameter the absolute values of relative prediction errors were considered (Tab. 3).

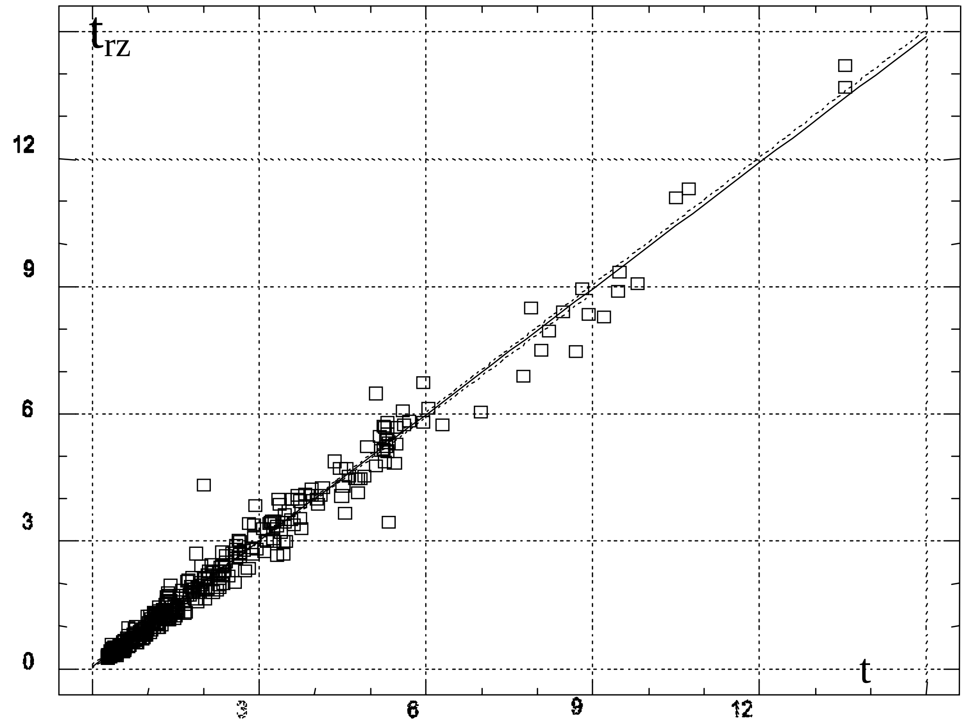

Figure 3: Regression of experimental freezing time vs. time calculated after new model

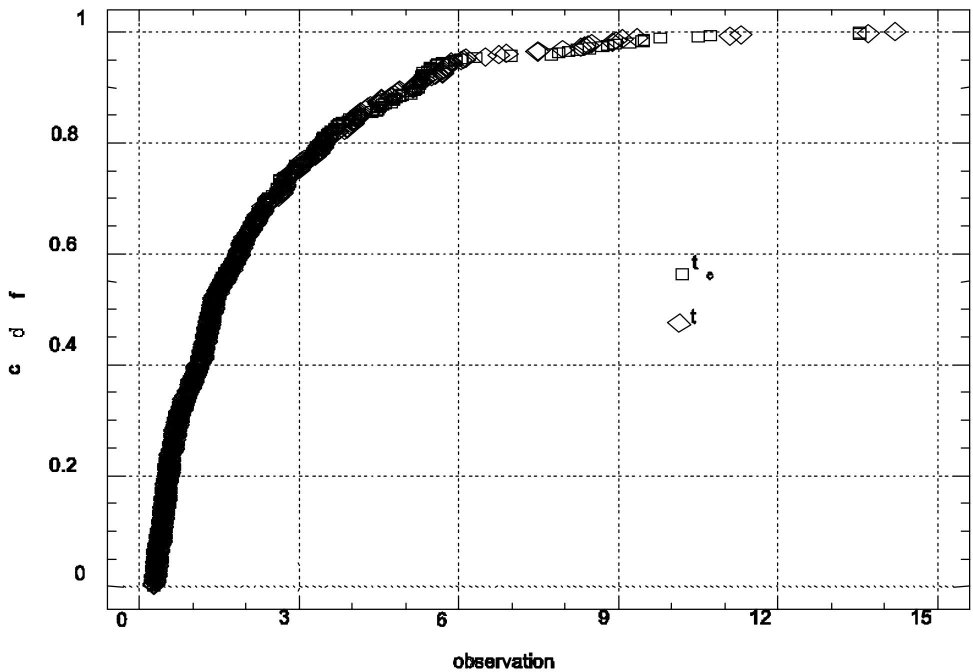

Figure 4: Plot of the cdf of experimental freezing time and time predicted after new model

Table 3: Means and confidence intervals for absolute values of prediction errors

	Sample size	Mean %	Std. Error	95% conf. int. for mean %
New model	383	9.83	0.399	8.923÷10.729
Pham	383	10.59	0.466	9.688÷11.493
Cleland	383	10.13	0.507	9.226÷11.031

Considering the thawing time prediction it can be assumed, that the best results are obtained after the models of: Cleland's (1986, 1987 EHTD, 1987 MCP) and the ours one. The above statement is confirmed by the data in Table 4.

Table 4: Descriptive statistics of thawing time prediction error *E* (sample size: 232)

	Mean	Median	Mode	Var.	SD	SE
Pham	21.4	23.03	20.0	530.7	23.04	1.51
Cleland	3.8	5.03	0	182.3	13.50	0.89
Cleland with EHTD	8.1	6.84	0	130.1	11.41	0.75
Cleland with MCP	7.9	6.40	0	134.3	11.59	0.76
This work	-3.2	-2.64	0	208.0	14.42	0.95

Regression analysis of the experimental thawing time against the values calculated after examined models leads to a statement, that the results after new model are good correlated (Tab. 5).

Table 5: Regression analysis of thawing time t_{rz} vs. the time calculated t

	This method	Cleland EHTD	Pham	Cleland MCP	Cleland
Intercept *A*	0,51	0,12	0,60	0,12	0,07
Slope *B*	0,88	0,91	0,74	0,91	0,94
R	0,9903	0,9980	0,9747	0,9978	0,9981

Summing up, it should be stated that in case of thawing time prediction after the studied models, the use of our new model and Cleland's models, guarantees the results approximating the real ones most.

Conclusions

1. The new simple calculation models for freezing and thawing time prediction were developed, which are characterised by a number of advantages including the fact that the calculations can be performed with a basic hand calculator.
2. Mathematical analysis of freezing and thawing time calculations confirmed high accuracy of the models. The mean relative error of freezing times prediction is 1.74%. In the case of new thawing time prediction model, low average value of error (-6,86%) was proved.

3. The regression equations of the freezing time in relation to the time determined by the new method indicates the significant correlation between both values (R = 0.9902). The same analysis on thawing time shows good agreement with experimental data (R = 0.9903).

4. The new models have got a constant introduced instead of initial cryoscopic temperature, therefore some potential errors can be omitted, that might result from inadequate values of this temperature placed to the model.

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